A novel approach to fixture design based on locating correctness

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Abstract: In a machining process, since the geometric accuracy of a manufacturing workpiece mainly relies on the relative position of workpiece to the machining tool, fixtures are needed to locate the workpiece. The clear concept of locating correctness is presented based on Venn diagram and a general algorithm is proposed to determine the locator number and layout. On one hand, the theoretical and practical constrained DOFs are formulated as a function of the machining requirement and the locating scheme, respectively. On the other hand, some criteria are concluded to analyse the locating correctness and modify the locating scheme with locating incorrectness.

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1 Introduction

During the machining operation, the function of a fixture is to ensure the location of the workpiece being manufactured to satisfy the manufacturing quality and locators are frequently used in contact with the workpiece to restrain related degrees of freedom. Therefore, the fixture-locating scheme, i.e., the locator number and layout, is needed to be correctly determined.

Traditional fixture designs were mainly based on some empirical principles (Hoffman, 1991). A drawing of fixture design is often prepared based on the processing plan and the workpiece information. Since the 1980s, the developments of Computer-Aided Fixture Design (CAFD) techniques have shortened greatly the time needed for fixture design and burdensome tasks such as consulting handbooks of jig and fixture design, drawing all assembly views and compiling a technical documentation have become greatly simplified. However, the fixture specification for the machining strongly depends on the designer’s experience and knowledge (Rong and Zhu, 1999).

For this reason, considerable efforts have been made to develop CAFD techniques. Representative works are as follows. Asada and By (1985) and Hong et al. (1996) developed a kinematic model of the locating scheme using Taylor expansion. The full-rank Jacobian matrix of the kinematic model is considered as a criterion to verify the complete location. Chou et al. (1989) and Trappey and Liu (1992) used the screw theory to formulate the fixturing equilibrium equation for a rigid prismatic workpiece. It concluded that the locating wrenches matrix of the fixturing equilibrium
equation needs to be full rank for the complete location. In fact, as the complete location is only one of the locating schemes being able to hold the workpiece in the desired position, the above-mentioned work was extended later by Kang et al. (2003) and Song and Rong (2005) to study non-deterministic location such as under- and over-locations. When the locating scheme is assumed to be correct, Wang (2002), Cai et al. (1997) and Qin et al. (2006) can optimise the locator dimensions and positions.

The above-mentioned investigations laid a good foundation for designing fixture-locating schemes. The locating correctness and the modification of incorrect locating scheme are, however, not addressed. This paper presents a new systematical approach to deal with the fixture design on the basis of the locating correctness.

2 Locating correctness

As shown in Figure 1, a free workpiece in the rectangular coordinate system can not only translate along $X$, $Y$ and $Z$ directions but also rotate about the $X$, $Y$ and $Z$ axes. The position variations caused by the workpiece movement are denoted by $\delta x_w$, $\delta y_w$, $\delta z_w$, $\delta \alpha_w$, $\delta \beta_w$ and $\delta \gamma_w$. This is the so-called DOFs.

**Figure 1** Six DOFs of a workpiece

2.1 Theoretically constrained DOFs

In order that the machining operation can be carried out correctly to obtain machining requirements, some DOFs of a workpiece must be constrained. These DOFs determined by the machining requirements are defined as the theoretically constrained DOFs.

Suppose a batch of workpieces will be milled at the top surface with design dimension $h$, as shown in Figure 2. In the machining process, the position of the cutting tool with respect to the $XZ$ plane is set up according to $h$ in advance. As the motion path of the cutting tool is planned for the machining operation, the machining dimension is evaluated according to the distance between the cutting tool and the bottom plane. As illustrated in Figure 3, the theoretical DOFs are easily known to be $\delta y_w$, $\delta \alpha_w$ and $\delta \beta_w$. Other three DOFs may not be constrained because the machining dimensions $h1$, $h3$ and $h5$ are equal to $h$. 
2.2 Practically constrained DOFs

During a machining operation, the theoretically constrained DOFs are constrained by fixture-locating scheme. Therefore, it is crucial for design dimensions to configure a feasible locating scheme. In principle, design of a locating scheme is to lay out a certain number of locators on the workpiece. A variety of locating schemes will be obtained by changing the number and layout of locators as shown in Figures 4–6.
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Figure 4  The practically constrained DOFs of scheme 1: (a) locating scheme 1 and (b) translation along the Z direction (see online version for colours)

![Diagram](image)

(a) (b)

Figure 5  The practically constrained DOFs of scheme 2: (a) locating scheme 2; (b) translation along the Z direction and (c) rotation about the Z axis (see online version for colours)

![Diagram](image)

(a) (b) (c)

Figure 6  Other locating schemes: (a) scheme 3; (b) scheme 4; (c) scheme 5 and (d) scheme 6 (see online version for colours)

![Diagram](image)

(a) (b) (c) (d)

Figure 4 shows a locating scheme with three locators on the bottom surface and two on the left surface. Thus, the workpiece is only movable along the Z direction.
The locating scheme shown in Figure 5 has two locators on the bottom and left surface, respectively. In this case, the workpiece is obviously free to either translate along the Z direction or rotate about the Z axis when locators 1 and 2 are arranged in parallel with the Z axis. This means that different locating schemes can constrain different DOFs. From the above-mentioned analyses, we know that locating scheme 1 constrains \( \delta_{xw}, \delta_{yw}, \delta_{z}, \delta_{\alpha}, \delta_{\beta}, \delta_{\gamma} \) whereas locating scheme 2 eliminates \( \delta_{xw}, \delta_{yw}, \delta_{\alpha}, \delta_{\beta} \). Here, the DOFs constrained by the locating scheme are called practically constrained DOFs.

By analogy, we can also investigate the practically constrained DOFs determined by other locating schemes shown in Figure 6. In cases of Figure 6(a) and (b), the practically constrained DOFs are \( \delta_{xw}, \delta_{yw}, \delta_{zw}, \delta_{\alpha}, \delta_{\beta}, \delta_{\gamma} \). The practically constrained DOFs of Schemes 5 and 6 are the same as in Schemes 1 and 2.

### 2.3 Logical relationship

Suppose \( \{ \delta_{{q}^*} \} \) and \( \{ \delta_{q} \} \) denote the set of theoretically constrained DOFs and practically constrained DOFs, respectively. \( \{ I \} = \{ \delta_{xw}, \delta_{yw}, \delta_{zw}, \delta_{\alpha}, \delta_{\beta}, \delta_{\gamma} \} \) is the full set. Obviously, \( \{ \delta_{{q}^*} \} \) and \( \{ \delta_{q} \} \) are the subsets of \( \{ I \} \). Thus, the relationship between \( \{ \delta_{{q}^*} \} \) and \( \{ \delta_{q} \} \) can be obtained by Venn diagram and categorised into including relationship, included relationship, intersection and difference, as shown in Figure 7.

**Figure 7** Venn diagram: (a) including relationship; (b) included relationship; (c) intersection and (d) difference

To obtain the specific position of the workpiece with respect to the cutting tool, the theoretically constrained DOFs should be reasonably identified. Therefore, \( \{ \delta_{{q}^*} \} \) must be included in \( \{ \delta_{q} \} \), as shown in Figure 7(b). In addition, if the number of \( \{ \delta_{{q}^*} \} \) is equal to the locator number, the locating correctness in this case can be ensured.

By comparing the practically constrained DOFs shown in Figures 4–6 with the theoretically constrained DOFs illustrated in Figure 2, schemes 1 and 3 are characteristic of locating correctness.
3 Algorithm of fixture design

It is important to analyse the locating correctness for the fixture design. When the locating correctness is valid, the designed fixture can be used to determine the position of the workpiece with respect to the cutting tool. Otherwise, it must be iteratively modified. Here, we propose a fixture design flow chart consisting of four parts. Each part is thought of as a mapping relationship or an algorithm, as shown in Figure 8. Part I refers to DOFs principle used to determine the theoretically constrained DOFs according to the machining requirements of the workpiece. Part II refers to locating principle determining the practically constrained DOFs based on the known locator number and layout. Part III refers to analytical criteria to verify the locating correctness by analysing theoretically constrained DOFs derived from machining requirements with respect to practically constrained DOFs. Part IV refers to modification criteria used to identify the cause of locating incorrectness for the eventual design modifications of the locating scheme.

**Figure 8** Fixture design flow chart based on locating correctness

![Fixture design flow chart](image)

3.1 DOFs principle

When a workpiece is free in 3D space, three linear velocities \( v_x, v_y, \) and \( v_z \), and three angular velocities \( \omega_x, \omega_y, \) and \( \omega_z \) can be used to describe the workpiece motion, as shown in Figure 9.
It is well known that both the position and the dimension accuracies of a machining surface depend on the fixture design. In fact, $\omega = [\omega_x, \omega_y, \omega_z]^T$ causes the orientation errors whereas both $v = [v_x, v_y, v_z]^T$ and $\omega$ produce the location errors or dimension errors. Here, the position error is divided into the orientation error (such as the parallelism error and the perpendicularity error) and the location error (such as the coaxiality error and the symmetry error).

Let $r_p = [x_p, y_p, z_p]^T$ denote the coordinates of an arbitrary point $P$ in the global coordinate system $\{XYZ\}$. Thus, the orientation error can be described as

$$v_{po} = \omega \times r_p.$$  

(1)

To study the relationship between the workpiece DOFs and its machining requirement, equation (1) is rewritten in differentiation form

$$\begin{bmatrix}
\delta x_{p0} \\
\delta y_{p0} \\
\delta z_{p0}
\end{bmatrix} =
\begin{bmatrix}
\omega_x z_p - \omega_z y_p \\
\omega_y x_p - \omega_z z_p \\
\omega_z y_p - \omega_y z_p
\end{bmatrix} =
\begin{bmatrix}
\delta \beta_x z_p - \delta \gamma_y y_p \\
\delta \gamma_y x_p - \delta \alpha_z z_p \\
\delta \alpha_z y_p - \delta \beta_y z_p
\end{bmatrix}$$  

(2)

where \( \delta q = [v^T, \omega^T] = [\delta x, \delta y, \delta z, \delta \alpha_x, \delta \beta_x, \delta \gamma_y]^T \) represents the desired DOF vector of the workpiece to be constrained correspondingly.

In addition, according to the velocity composition law of a kinematic particle, the absolute velocity $v_p$ is the vector sum of the relative velocity $v_o$ and the transportation velocity $v_{po}$ of point $P$. The following equation can be obtained as

$$v_p = v_o + v_{po}$$  

(3)

with

$$v_o = v_o.$$  

(4)

The substitution of equations (1) and (4) into equation (3) produces the location errors or dimension errors written as
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\[ \begin{bmatrix} \delta x_p \\ \delta y_p \\ \delta z_p \end{bmatrix} = \begin{bmatrix} \delta x_u + \delta \beta_z p_x - \delta \gamma_y y_p \\ \delta y_u + \delta \gamma_x x_p - \delta \alpha_z z_p \\ \delta z_u + \delta \alpha_y y_p - \delta \beta_z x_p \end{bmatrix} \]  \tag{5}

It is worth noticing that the above-mentioned relation is valid only when point \( P \) is selected as a point on the reference related to the machining requirements. According to equations (2) and (5), only when some or all six DOFs of the workpiece are constrained, the machining accuracy can be ensured.

For example, if the machining dimension is aligned in \( X \) direction, the following equation can be obtained from equation (5),

\[ \delta x_p = \delta x_u + \delta \beta_z p_x - \delta \gamma_y y_p = 0. \]  \tag{6}

Because \( y_p \) and \( z_p \) can take arbitrary values, equation (6) is valid if and only if

\[ \delta x_u = \delta \beta_z = \delta \gamma_y = 0. \]  \tag{7}

In other words, only when three DOFs \( \delta x_u, \delta \beta_z, \delta \gamma_y \) are constrained, the machining operation is correct. Thus

\[ \delta q^*_u = [0, \lambda, \lambda, \lambda, 0, 0]^T \]  \tag{8}

where \( \lambda, \lambda, \lambda \) are arbitrary numbers.

3.2 Locating principle

As shown in Figure 10, the locating scheme consists of \( k \) \((i = 1, 2, \ldots, k)\) locators. Suppose that the workpiece is a rigid body with a contact surface described by a piecewise differentiable function in the workpiece coordinate system \( \{X^wY^wZ^w\} \). If \( \mathbf{r}_i^w = [x_i^w, y_i^w, z_i^w]^T \) designates the coordinate vector of \( i \)th contact point on the workpiece with respect to \( \{X^wY^wZ^w\} \), the contact surface of the workpiece is represented as

\[ f(\mathbf{r}_i^w) = 0. \]  \tag{9}

If the orientation and position of the workpiece are known, the \( i \)th contact point \( \mathbf{r}_i^w \) can be mapped from \( \{X^wY^wZ^w\} \) to \( \{XYZ\} \) following

\[ \mathbf{r}_i = \mathbf{T}(\Theta_u) \mathbf{r}_i^w + \mathbf{r}_u \]  \tag{10}

where \( \mathbf{r}_u = [x_u, y_u, z_u]^T \) denotes the position of the origin of \( \{X^wY^wZ^w\} \) in \( \{XYZ\} \). \( \Theta_u = [\alpha_u, \beta_u, \gamma_u]^T \) is the orientation of \( \{X^wY^wZ^w\} \) with respect to \( \{XYZ\} \). And

\[ \mathbf{T}(\Theta_u) = \begin{bmatrix} c\beta_u c\gamma_u & -ca_u s\gamma_u + sa_u s\beta_u c\gamma_u & sa_u s\gamma_u + ca_u s\beta_u c\gamma_u \\ c\beta_u s\gamma_u & ca_u c\gamma_u + sa_u s\beta_u s\gamma_u & -sa_u c\gamma_u + ca_u s\beta_u s\gamma_u \\ -s\beta_u & ca_u c\beta_u & ca_u s\beta_u \end{bmatrix} \]  \tag{11}

is an orthogonal rotation matrix with \( c = \cos \) and \( s = \sin \).
Let \( \mathbf{r}_i = [x_i, y_i, z_i]^T \) (1 \( \leq i \leq k \)) be the coordinate vector of the \( i \)th contact point in \{XYZ\}, then the following equation can be obtained by substituting equation (10) into equation (9)

\[
f(q_w, r_i) = f[T(\Theta)_w]^T (\mathbf{r}_j - \mathbf{r}_i)] = 0
\]

where \( q_w = [r_w^T, \Theta_w^T]^T \) denotes the workpiece position and orientation.

Without loss of generality, \{\textit{X}\textsuperscript{w}\textit{Y}\textsuperscript{w}\textit{Z}\textsuperscript{w}\} and \{XYZ\} can be always assumed to have the identical orientation. Thus, the difference of equation (12) with respect to \( q_w \) and \( r_i \) leads to

\[
\mathbf{J} \delta q_w = \mathbf{0}
\]

where \( \delta q_w \) is the so-called practically constrained DOFs of the workpiece depending on the locator number \( k \) and layout \( r_i \) and \( n_i \), \( 1 \leq i \leq k \).

\[
\mathbf{J} = \begin{bmatrix}
\mathbf{n}_1 & \mathbf{n}_2 & \cdots & \mathbf{n}_k \\
\mathbf{n}_1 \times \mathbf{r}_i & \mathbf{n}_2 \times \mathbf{r}_i & \cdots & \mathbf{n}_k \times \mathbf{r}_i
\end{bmatrix}
\]

is the locating Jacobian matrix described in Appendix A.

\[
\mathbf{n}_i = \frac{\partial f}{\partial \mathbf{r}_i} = \begin{bmatrix}
\frac{\partial f}{\partial x_i} & \frac{\partial f}{\partial y_i} & \frac{\partial f}{\partial z_i}
\end{bmatrix}^T, \quad 1 \leq i \leq k
\]

is the \( i \)th unit normal vector of the workpiece surface at the \( i \)th contact point.

### 3.3 Analytical criteria

It is well known that six column vectors \( \xi_x = [1, 0, 0, 0, 0, 0]^T \), \( \xi_y = [0, 1, 0, 0, 0, 0]^T \), \( \xi_z = [0, 0, 1, 0, 0, 0]^T \), \( \xi_w = [0, 0, 0, 1, 0, 0]^T \), \( \xi_{\alpha} = [0, 0, 0, 0, 1, 0]^T \), and \( \xi_{\beta} = [0, 0, 0, 0, 0, 1]^T \) are the standard orthogonal bases in the linear six-dimension space. Thus, \( \delta q_w \) can be expressed as a linear combination of them. For example, the machining dimension related to equation (7) corresponds to

\[Figure 10\] Illustration of the locating scheme to the workpiece (see online version for colours)
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\[ \delta \mathbf{q}_w = 0 \xi_1 + \lambda_1 \xi_2 + \lambda_2 \xi_3 + \lambda_3 \xi_4 + 0 \xi_5 + 0 \xi_6 = \mathbf{\xi}. \]  

(16)

where \( \mathbf{\xi} = [\xi_1, \xi_2, \xi_3, \xi_4] \) is the base vector involved in \( \delta \mathbf{q}_w \). \( \mathbf{\lambda} = [\lambda_1, \lambda_2, \lambda_3] \) is the constant vector with arbitrary numbers \( \lambda_1, \lambda_2, \lambda_3 \).

Let \( \text{rank}(\mathbf{\xi}) \) denote the rank of the base vector involved in \( \delta \mathbf{q}_w \), the number of theoretically unconstrained DOFs in \( \delta \mathbf{q}_w \) can be known to be \( \text{rank}(\mathbf{\xi}) \). If \( m \) vector bases in \( \delta \mathbf{q}_w \) are not the solutions of equation (13), \( \text{rank}(\mathbf{\xi}) - m \) is then the number of DOFs belonging to the practically constrained DOFs. Some analytical criteria can be derived here to verify the locating correctness.

**Criterion 1:** If \( k < 6 \), then the locating scheme is

- partial location when \( \text{rank}(\mathbf{J}) = k \) and \( \text{rank}(\mathbf{J}) + \text{rank}(\mathbf{\xi}) - m = 6 \), as shown in Figure 4(a)
- under-location when \( \text{rank}(\mathbf{J}) = k \) and \( \text{rank}(\mathbf{J}) + \text{rank}(\mathbf{\xi}) - m < 6 \), as shown in Figure 5(a)
- partial over-location when \( \text{rank}(\mathbf{J}) < k \) and \( \text{rank}(\mathbf{J}) + \text{rank}(\mathbf{\xi}) - m = 6 \)
- under over-location when \( \text{rank}(\mathbf{J}) < k \) and \( \text{rank}(\mathbf{J}) + \text{rank}(\mathbf{\xi}) - m < 6 \), as shown in Figure 6(d).

**Criterion 2:** If \( k = 6 \), then the locating scheme is

- complete location when \( \text{rank}(\mathbf{J}) = 6 \), as shown in Figure 6(a)
- partial over-location when \( \text{rank}(\mathbf{J}) < 6 \) and \( \text{rank}(\mathbf{J}) + \text{rank}(\mathbf{\xi}) - m = 6 \), as shown in Figure 6(c)
- under over-location when \( \text{rank}(\mathbf{J}) < 6 \) and \( \text{rank}(\mathbf{J}) + \text{rank}(\mathbf{\xi}) - m < 6 \).

**Criterion 3:** If \( k > 6 \), then the locating scheme is

- complete over-location when \( \text{rank}(\mathbf{J}) = 6 \), as shown in Figure 6(b)
- partial over-location when \( \text{rank}(\mathbf{J}) < 6 \) and \( \text{rank}(\mathbf{J}) + \text{rank}(\mathbf{\xi}) - m = 6 \)
- under over-location when \( \text{rank}(\mathbf{J}) < 6 \) and \( \text{rank}(\mathbf{J}) + \text{rank}(\mathbf{\xi}) - m < 6 \).

Finally, the flow chart is given in Figure 11 to summarise judgement criteria for different locating schemes. It is worth noticing that both complete location and partial location belong to correct locating scheme whereas neither under-location nor under over-location are viable. Generally speaking, though the application of the great number of locators can increase the workpiece stiffness, the addition of a locator into a fixture system may increase the workpiece set-up time, fixture capital cost and weight, the complexity of cutting tool access to the workpiece. From the viewpoint of engineering application, the locating scheme being complete over-location or partial over-location is thought to be incorrect. Therefore, the locator number is not more than six.
3.4 Modification criteria

Generally speaking, a workable locating scheme is able to constrain the undesired DOFs of the workpiece. The fixture designer must determine the layout of a given number of locators so as to locate the workpiece in its desired position. In fact, a variety
of locating schemes will be obtained by changing the number and layout of locators. According to the three criteria proposed in the above-mentioned section, we can analyse the correctness of the locating scheme. However, it is more important to figure out incorrect locating schemes. To do this, three additional criteria are given to analyse the number and layout of locators.

**Criterion 4:** If \( k < 6 \), then
- when \( \text{rank}(\mathbf{J}) = k \) and \( \text{rank}(\mathbf{J}) + \text{rank}(\mathbf{\xi}) - m < 6 \), the locator number is insufficient
- when \( \text{rank}(\mathbf{J}) < k \) and \( \text{rank}(\mathbf{J}) + \text{rank}(\mathbf{\xi}) - m = 6 \), the locator layout is incorrect
- when \( \text{rank}(\mathbf{J}) < k \) and \( \text{rank}(\mathbf{J}) + \text{rank}(\mathbf{\xi}) - m < 6 \), the locator layout is improper.

**Criterion 5:** If \( k = 6 \), then
- when \( \text{rank}(\mathbf{J}) < k \) and \( \text{rank}(\mathbf{J}) + \text{rank}(\mathbf{\xi}) - m = 6 \), the locator layout is unreasonable
- when \( \text{rank}(\mathbf{J}) < k \) and \( \text{rank}(\mathbf{J}) + \text{rank}(\mathbf{\xi}) - m < 6 \), the locator layout is infeasible.

**Criterion 6:** If \( k > 6 \), then
- when \( \text{rank}(\mathbf{J}) = 6 \), the locator number is usually overabundant
- when \( \text{rank}(\mathbf{J}) < 6 \) and \( \text{rank}(\mathbf{J}) + \text{rank}(\mathbf{\xi}) - m = 6 \), the locator number is superabundant
- when \( \text{rank}(\mathbf{J}) < 6 \) and \( \text{rank}(\mathbf{J}) + \text{rank}(\mathbf{\xi}) - m < 6 \), not only the locator number is excessive, but also the locator layout is unacceptable.

### 4 Numerical tests

Here, two numerical examples are used to demonstrate the procedure of fixture design shown in Figure 11. First, the theoretically constrained DOFs and the practically constrained DOFs are evaluated by means of equations (2) and (5) and equation (13), respectively. And then, the locating correctness is analysed according to Criteria 1–3. In the wrong case, a redesign will be carried out by means of Criteria 4–6.

#### 4.1 “One-plane-two-pin” locating scheme

As shown in Figure 12, the workpiece is located to drill eight through holes of dimension \( d = 0.01_{+0.10} \) mm. The diameter of the workpiece is \( d = 0.06240 \) mm. The eight holes are uniformly distributed along the circle of diameter \( d = 30 \) mm. The fixture-locating scheme consists of two cylindrical pins and one support plate. The diameters of two cylinders are \( D_1 = D_2 = 20^{0.10}_{-0.02} \) mm with a span of \( B = 35 \pm 0.01 \) mm after the set-up.

In this case, the contact between each locator and the workpiece can be simplified as point contact. Coordinates of all contact points (i.e., locating points) and their normal unit vectors are listed in Table 1.
Figure 12 "One-plane-two-pin" locating scheme (see online version for colours)

Table 1 Positions and orientations of contact points

<table>
<thead>
<tr>
<th>Locators</th>
<th>Locating points</th>
<th>Positions</th>
<th>Unit normal vectors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Support plate</td>
<td>1 ([x_1, y_1, 0]^T)</td>
<td>([0, 0, 1]^T)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2 ([x_2, y_2, 0]^T)</td>
<td>([0, 0, 1]^T)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3 ([x_3, y_3, 0]^T)</td>
<td>([0, 0, 1]^T)</td>
<td></td>
</tr>
<tr>
<td>Cylindrical pin</td>
<td>4 ([-1.1666, -1.6244, z_4]^T)</td>
<td>([0.5833, 0.8122, 0]^T)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5 ([1.1666, -1.6244, z_5]^T)</td>
<td>([-0.5833, 0.8122, 0]^T)</td>
<td></td>
</tr>
</tbody>
</table>

On the basis of the locating correctness, the fixture design is performed in the following steps.

*Determine the theoretically constrained DOFs*

The hole diameter \(d_1\) is ensured by the diameter of cutting tool. However, the circle of diameter \(d_2\) along which eight holes are uniformly distributed is guaranteed by the fixture. As shown in Figure 12, because the reference of the diameter \(d_2\) is Z axis, the following conditions exist
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\[ x_p = y_p = 0. \] \hfill (17)

In addition, because the diameter \( d_2 \) is in the arbitrary directions, another condition has to hold according to equation (5),

\[
\begin{align*}
\delta x_p &= \delta x_u + \delta \beta_z z_p - \delta \gamma_{x} y_p = 0 \\
\delta y_p &= \delta y_u + \delta \gamma_x x_p - \delta \alpha_z z_p = 0
\end{align*}
\hfill (18)
\]

with the arbitrary number \( z_p \).

By combining equation (18) with equation (17), the theoretically constrained DOFs are then obtained as

\[ \delta x_u = \delta y_u = \delta \alpha_u = \delta \beta_u = 0. \] \hfill (19)

Therefore, we have

\[ \delta \mathbf{q}_\nu = \lambda \xi + \lambda \gamma \xi. \] \hfill (20)

**Determine the practically constrained DOFs**

The locating Jacobian matrix can be first calculated according to equation (14), i.e.,

\[
\mathbf{J} = \\
\begin{bmatrix}
0 & 0 & -1 & y_1 & -x_1 & 0 \\
0 & 0 & -1 & y_2 & -x_2 & 0 \\
0 & 0 & -1 & y_3 & -x_3 & 0 \\
-0.5833 & -0.8122 & 0 & -0.8122 z_4 & 0.5833 z_4 & 0 \\
0.5833 & -0.8122 & 0 & -0.8122 z_5 & -0.5833 z_5 & 0
\end{bmatrix}. \hfill (21)
\]

And then, by substituting two bases (\( \xi \) and \( \xi \)) into equation (13), the following equations can be derived as

\[
\mathbf{J} \xi = \begin{bmatrix} 0, 0, 1, 0, 0, 0 \end{bmatrix}^T \neq \mathbf{0} \hfill (22)
\]

\[ \mathbf{J} \xi = \mathbf{0}. \hfill (23) \]

Obviously, \( \xi \) is not a solution of equation (13) so that

\[ m = 1. \] \hfill (24)

In fact, when equation (13) is solved, we can determine the practically constrained DOFs as

\[ \delta \mathbf{q}_u = \xi, \lambda. \] \hfill (25)

**Analyse the locating correctness**

According to equations (19)–(23), we can obtain the following value,

\[ \text{rank} (\mathbf{J}) + \text{rank}(\xi) - m = 5 + 2 - 1 = 6. \] \hfill (26)

As equation (26) belongs to criterion 1, such a locating scheme is hence identified to be a partial location so that it can ensure the machining dimension \( d_2 \).
4.2 “3-2-1” locating scheme

As shown in Figure 13, a fixture configuration is designed to mill a through step on the workpiece. The workpiece is held by six locators L1, L2, L3, L4, L5 and L6 whose coordinates and normal unit vectors are given in Table 2.

![Figure 13 “3-2-1” locating scheme: (a) before modified and (b) after modified (see online version for colours)](image)

<table>
<thead>
<tr>
<th>Locator</th>
<th>Before modified</th>
<th>After modified</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1</td>
<td>([x_1, 0, z_1]^T)</td>
<td>([x_1, 0, z_1]^T)</td>
</tr>
<tr>
<td></td>
<td>([0, 1, 0]^T)</td>
<td>([0, 1, 0]^T)</td>
</tr>
<tr>
<td>L2</td>
<td>([x_2, 0, z_2]^T)</td>
<td>([x_2, 0, z_2]^T)</td>
</tr>
<tr>
<td></td>
<td>([0, 1, 0]^T)</td>
<td>([0, 1, 0]^T)</td>
</tr>
<tr>
<td>L3</td>
<td>([x_3, 0, z_3]^T)</td>
<td>([x_3, 0, z_3]^T)</td>
</tr>
<tr>
<td></td>
<td>([0, 1, 0]^T)</td>
<td>([0, 1, 0]^T)</td>
</tr>
<tr>
<td>L4</td>
<td>([0, y_4, z_4]^T)</td>
<td>([0, y_4, z_4]^T)</td>
</tr>
<tr>
<td></td>
<td>([1, 0, 0]^T)</td>
<td>([1, 0, 0]^T)</td>
</tr>
<tr>
<td>L5</td>
<td>([0, y_5, z_4]^T)</td>
<td>([0, y_5, z_4]^T)</td>
</tr>
<tr>
<td></td>
<td>([1, 0, 0]^T)</td>
<td>([1, 0, 0]^T)</td>
</tr>
<tr>
<td>L6</td>
<td>([x_6, y_6, 0]^T)</td>
<td>([x_6, y_6, 0]^T)</td>
</tr>
<tr>
<td></td>
<td>([0, 0, 1]^T)</td>
<td>([0, 0, 1]^T)</td>
</tr>
</tbody>
</table>

Table 2 Positions and orientations of six locators
To modify the fixture locating scheme, the procedure is as follows:

**Determine the theoretically constrained DOFs**

In Figure 13(a), \( h \) and \( l \) denote machining dimensions along \( Y \) and \( Z \) directions, respectively. According to equation (5), the following conditions have to hold,

\[
\begin{align*}
\delta y_p &= \delta y_w + \delta \gamma_w x_p - \delta \alpha_w z_p = 0 \\
\delta z_p &= \delta z_w + \delta \alpha_w y_p - \delta \beta_w x_p = 0.
\end{align*}
\]  

(27)

The theoretically constrained DOFs are then

\[
\delta y_w = \delta z_w = \delta \alpha_w = \delta \beta_w = \delta \gamma_w = 0.
\]  

(28)

Therefore, we have

\[
\delta q^*_w = \lambda \xi_w.
\]  

(29)

**Determine the practically constrained DOFs**

In Figure 13(a) and Table 2, the substitution of \( \xi_w \) into equation (13) gives rise to

\[
J \xi_w = \begin{bmatrix}
0 & -1 & 0 & -z_4 & x_4 \\
0 & -1 & 0 & -z_2 & x_2 \\
0 & -1 & 0 & -z_3 & x_3 \\
-1 & 0 & 0 & 0 & z_4 \\
-1 & 0 & 0 & 0 & z_3 \\
0 & 0 & -1 & y_4 & -x_4
\end{bmatrix}
= \begin{bmatrix}
1 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix} \neq 0.
\]  

(30)

Obviously, \( \xi_w \) is not a solution of equation (13) so that

\[
m = 1.
\]  

(31)

In addition, equation (13) can be solved to obtain the practically constrained DOFs as

\[
\delta q_w = \begin{bmatrix}
-z_4 \\
x_4 \\
0 \\
1 \\
0
\end{bmatrix} \lambda
\]  

with the arbitrary number \( \lambda \).

Obviously, the practically constrained DOFs are as follows

\[
\delta y_w = \delta \alpha_w = \delta \gamma_w = 0.
\]  

(33)

**Analyse the locating correctness**

According to equations (29)–(31), we can obtain the following value,

\[
\text{rank}(J) + \text{rank}(\xi) - m = 5 + 1 - 1 = 5 < 6.
\]  

(34)
By means of criterion 2, such a locating scheme is identified to be an under over-location, which is an infeasible locating scheme and cannot guarantee the machining dimension $l$ and $h$ of the workpiece.

**Modify the incorrect locating scheme**

Again, according to criterion 5, we can know that both the locator layout and the number are unreasonable. To obtain the machining dimension $l$ and $h$, the layout of locators has to be redesigned as that in Figure 13(b) and Table 2. As a result, the following relation can be easily obtained as

$$\text{rank}(J) = 6 = k.$$  (35)

Finally, the modified locating scheme belongs to an applicable complete location of criterion 2 being able to determine correctly the workpiece position and orientation.

### 5 Conclusions

A novel locating correctness-based methodology is developed for fixture analysis and design. First, based on the velocity composition law of a particle movement, the DOFs of the workpiece to be essentially constrained are determined following machining requirements. Second, practically constrained DOFs are formulated as a function of the locator number and layout. Finally, quantitative criteria are concluded to capture the relationship between the theoretically constrained DOFs and the practically constrained DOFs. The proposed approach is able to verify the locating correctness and to figure out the cause of locating incorrectness. It provides a practical tool for the development of CAFD system and enriches the quantitative formulation theory of fixture design.

### Acknowledgements

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### References


A novel approach to fixture design based on locating correctness


Appendix A: Locating Jacobian matrix

To study the determination of the locating Jacobian matrix, equation (12) is, respectively, differentiated with respect to \( x_w, y_w, z_w, \alpha_w, \beta_w \) and \( \gamma_w \) such that

\[
\begin{align*}
\frac{\partial f(q_w, r_w)}{\partial x_w} &= -\left[ \frac{\partial f(q_w, r_w)}{\partial r_1} \right]^T \frac{\partial r_1}{\partial x_w} \\
\frac{\partial f(q_w, r_w)}{\partial y_w} &= -\left[ \frac{\partial f(q_w, r_w)}{\partial r_1} \right]^T \frac{\partial r_1}{\partial y_w} \\
\frac{\partial f(q_w, r_w)}{\partial z_w} &= -\left[ \frac{\partial f(q_w, r_w)}{\partial r_1} \right]^T \frac{\partial r_1}{\partial z_w} \\
\frac{\partial f(q_w, r_w)}{\partial \alpha_w} &= \left[ \frac{\partial f(q_w, r_w)}{\partial r_1} \right]^T \frac{\partial \left[ T(\Theta_w)^T \right]}{\partial \alpha_w} r_i \\
\frac{\partial f(q_w, r_w)}{\partial \beta_w} &= \left[ \frac{\partial f(q_w, r_w)}{\partial r_1} \right]^T \frac{\partial \left[ T(\Theta_w)^T \right]}{\partial \beta_w} r_i \\
\frac{\partial f(q_w, r_w)}{\partial \gamma_w} &= \left[ \frac{\partial f(q_w, r_w)}{\partial r_1} \right]^T \frac{\partial \left[ T(\Theta_w)^T \right]}{\partial \gamma_w} r_i 
\end{align*}
\]

(36)

It is known from equation (11) that \( T(\Theta_w) \) is a function of \( \alpha_w, \beta_w \) and \( \gamma_w \), the differential of \( T(\Theta_w) \) can easily be obtained as
Besides, as \( \mathbf{r}_c = [x_c, y_c, z_c]^T \) is the position of the origin of \( \{X^wY^wZ^w\} \) in \( \{XYZ\} \), the following relations can be easily derived

\[
\frac{\partial \mathbf{r}_c}{\partial x_w} = [1, 0, 0]^T
\]
\[
\frac{\partial \mathbf{r}_c}{\partial y_w} = [0, 1, 0]^T.
\]
\[
\frac{\partial \mathbf{r}_c}{\partial z_w} = [0, 0, 1]^T
\]

Without loss of generality, \( \{X^wY^wZ^w\} \) and \( \{XYZ\} \) can be always assumed to have the identical orientation. By substituting equations (37) and (38) into equation (36), the \( i \)th row of Jacobian matrix can be obtained as

\[
\mathbf{J}_{ii} = -[\mathbf{n}_i, (\mathbf{n}_i \times \mathbf{r}_c)]^T.
\]

When \( k \) locating points exist in the locating scheme, the locating Jacobian matrix can be expressed as equation (14).